

Quantitative properties of sovereign default models: solution methods matter Technical appendix

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Abstract

This document describes how we evaluate the accuracy of the solution of the baseline sovereign default model using the test proposed by den Haan and Marcet (1994). We show that the solutions obtained using Chebyshev collocation and cubic spline interpolation approximate the equilibrium with reasonable accuracy and illustrate the challenges that arise when the test is applied to the solution obtained using the discrete state space technique.

Implementation of den Haan and Marcet (1994)

In order to save on notation, consider the case in which the shocks affect only the endowment level, as in Arellano (2008). In a differentiable problem, the Euler equation can be written as

$$\beta \int [1 - d(b', y')] u_1(c(b', y')) F(dy' | y) - u_1(c(b, y)) [q(b', y) + b' q_1(b', y)] = 0 \quad (1)$$

for all states in which the agent is not excluded from capital markets. The function d denotes the optimal default rule and takes a value of 1 (0) when the agent finds it optimal to default (repay the debt). The function c denotes the optimal consumption rule. When the agent is not excluded from capital markets c satisfies

$$c(b, y) = y + b - b'(b, y)q(b'(b, y), y).$$

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Following den Haan and Marcet (1994), let u_{t+1} denote the residual in the Euler equation at date $t + 1$, namely

$$u_{t+1} = \beta [1 - d(b_{t+1}, y_{t+1})] u_1(c(b_{t+1}, y_{t+1})) - u_1(c(b_t, y_t)) [q(b_{t+1}, y_t) + b_{t+1}q_1(b_{t+1}, y_t)]. \quad (2)$$

If the numerical solution is accurate, the residual u_{t+1} satisfies

$$E[u_{t+1} \otimes h(x_t)] = 0 \quad (3)$$

for any k -dimensional vector x_t that includes current and past values of state variables and any function $h : R^k \rightarrow R^p$.

Under the null hypothesis that equation (3) holds, the probability distribution of the den Haan and Marcet's statistic tends to a χ^2 with p degrees of freedom.

We implemented the test using $h(x_t) = 1$ and $h(x_t) = [1, y_t, b_t]$. The statistic was computed using 5,000 samples of 1,500 periods each. We removed the first 10 periods of each sample, all periods in which the economy is excluded with the exception of periods in which a default is declared, and the first 10 periods after the end of an exclusion spell. We did not observe any significant changes if more initial periods were removed.

Chebyshev collocation and spline interpolation perform well

Figures 1-3 show the distribution of the den Haan and Marcet's statistic in the simulations of Aguiar and Gopinath (2006) and Arellano (2008) when the model is solved using Chebyshev collocation and spline interpolation, and when the residuals are weighted by $h(x_t) = [1, y_t, b_t]$. Figures 1 and 2 show that the distributions of the den Haan and Marcet's statistic in simulations of Aguiar and Gopinath (2006) are close to their theoretical distribution under the null that the Euler equation is satisfied. The fit is not as good in the right tail of the distribution for the case of Arellano (2008) (see Figure 3). In that case, the correlation between the residuals and the residuals weighted by the endowment realization in the previous period is close to 0.99. The high co-linearity between these two series may reduce the precision of the test. Figure 4 shows that the fit of the den Haan and Marcet's statistic in simulations of Arellano (2008) is almost perfect when the statistic is computed using $h(x_t) = 1$.

Problems with the implementation of the test when the model is solved using discrete state space

Figures 5-7 show the distribution of the den Haan and Marcet's statistic when the solution is obtained using discrete state space and the residuals are weighted by $h(x_t) = [1, y_t, b_t]$. The graphs show that the differences between the distribution of the statistic and its theoretical distribution under the null are large and do not necessarily diminish with the number of grid points. The fit is not better when the residuals are weighted by the function $h(x_t) = [1]$. One might conclude from Figures 5-7 that DSS does not approximate the solution with reasonable accuracy. However, we found evidence suggesting that the main reason for the large discrepancies illustrated in Figures 5-7 can be traced back to approximation errors in the calculation of the residuals of the Euler equation.

The issue that we identified as a likely cause of the problem is that the derivative $q_1(b', y)$ need not be well approximated when the problem is solved using the discrete state space technique.

Let $\vec{b} = (b_1, \dots, b_{N_b})$ denote the vector of grid points for assets and $\vec{y} = (y_1, \dots, y_{N_y})$ denote the vector of grid points for endowment shocks.

When the model is solved using the discrete state space technique, we use the following approximation for $q_1(b', y)$:

$$q_1(b_i, y_j) = \frac{q(b_{i+\Delta}, y_j) - q(b_{i-\Delta}, y_j)}{b_{i+\Delta} - b_{i-\Delta}}, \quad (4)$$

with $\Delta = 1$.¹

Figure 8 illustrates the nature of the distortions using Model I in Aguiar and Gopinath (2006), but the same issue is present for other parameterizations of the model. Figure 8 shows the menu of bond prices and the agent's optimal choices for the two finest grid configurations used in the paper. When the number of asset grid points is increased to 7,000 points, we concentrate the grid points within an intermediate range, as explained in Table 2 of the paper (page 10).

¹When the model is solved using Chebyshev collocation or spline interpolation the derivative is approximated as

$$q_1(b', y) = \frac{q(b' + \epsilon, y) - q(b' - \epsilon, y)}{2\epsilon},$$

with $\epsilon = 10^{-5}$.

Figure 8 shows how the solution obtained with the finest grid configuration may introduce a downward bias to $q_1(b', y)$ when the latter is computed using equation (4) with $\Delta = 1$. Figure 9 illustrates that the bias in the measurement of $q_1(b', y)$ may indeed be systematic. Any bias in the approximation of $q_1(b', y)$ introduces a bias in the residuals u_{t+1} and, as a result, equation (3) is statistically rejected.

Figure 10 provides further evidence of the distortions raised by different approximations of q_1 . Figure 10 shows the distribution of the den Haan and Marcet's statistic in simulations of Arellano (2008) when the model was computed using discrete state space and our finest grid configuration. Figure 10 shows that the fit of the den Haan and Marcet's statistic varies substantially with Δ .

References

- Aguiar, M. and Gopinath, G. (2006). 'Defaultable debt, interest rates and the current account'. *Journal of International Economics*, volume 69, 64–83.
- Arellano, C. (2008). 'Default Risk and Income Fluctuations in Emerging Economies'. *American Economic Review*, volume 98(3), 690–712.
- den Haan, W. J. and Marcet, A. (1994). 'Accuracy in simulations'. *Review of Economic Studies*, volume 61, 3–17.

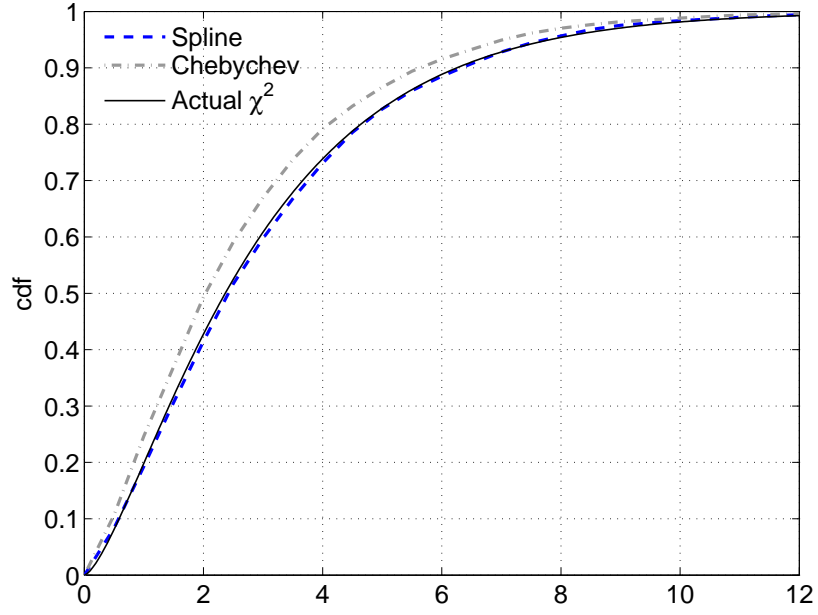


Figure 1: Cumulative distribution function of the den Haan and Marcet's statistic in Model I of Aguiar and Gopinath (2006) when the model is solved with Chebyshev collocation and spline interpolation and for weights $h(x_t) = [1, y_t, b_t]$.

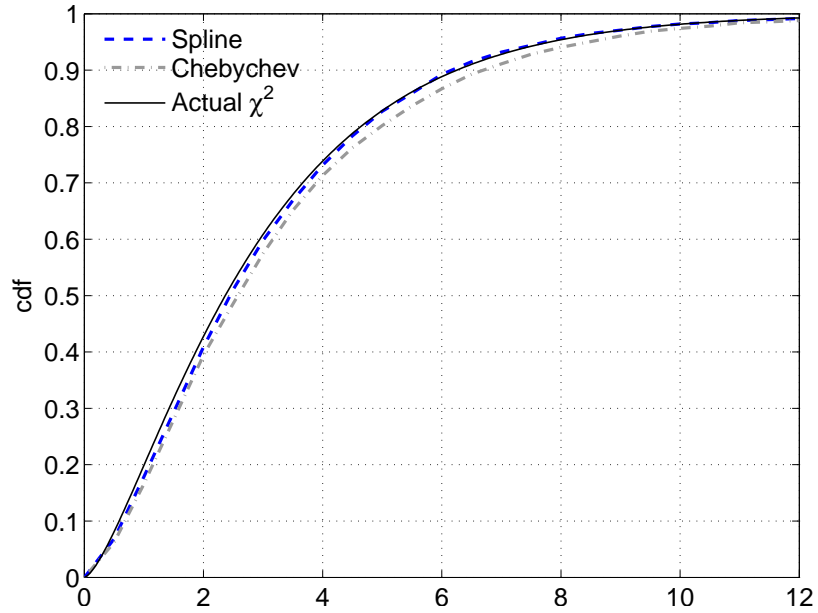


Figure 2: Cumulative distribution function of the den Haan and Marcet's statistic in Model II of Aguiar and Gopinath (2006) when the model is solved with Chebyshev collocation and spline interpolation and for weights $h(x_t) = [1, y_t, b_t]$.

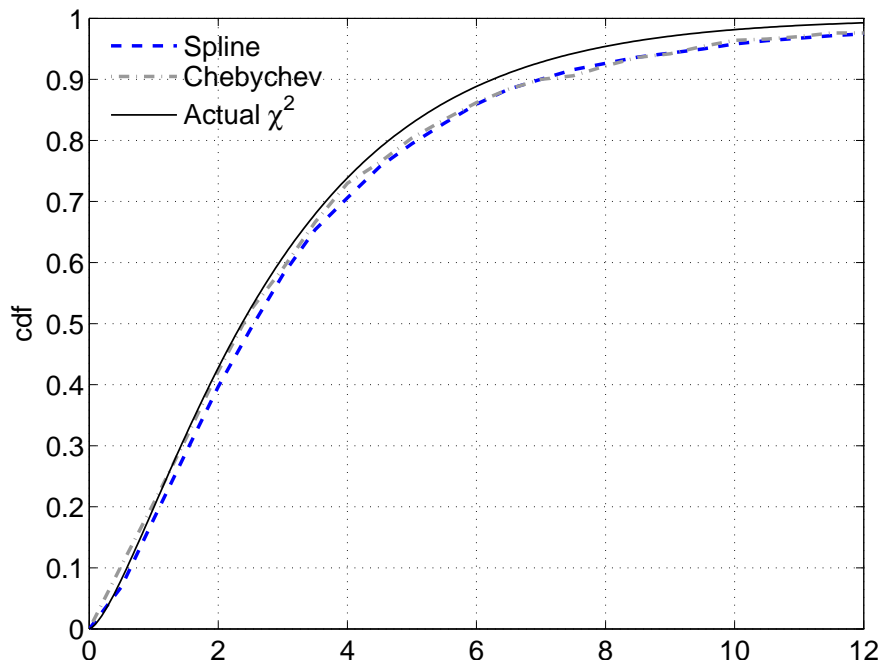


Figure 3: Cumulative distribution function of the den Haan and Marcet's statistic in Arellano (2008) when the model is solved with Chebyshev collocation and spline interpolation and for weights $h(x_t) = [1, y_t, b_t]$.

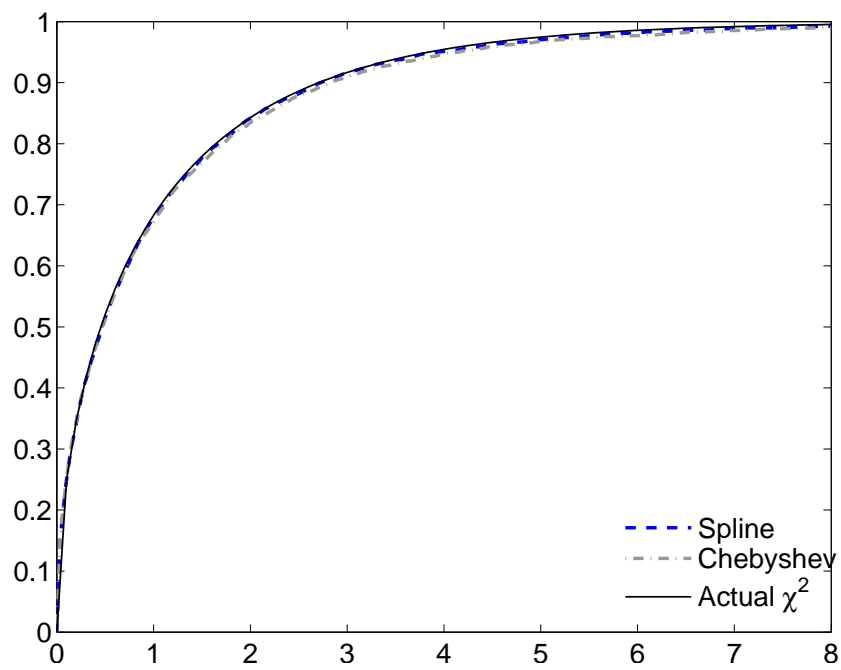


Figure 4: Cumulative distribution function of the den Haan and Marcet's statistic in Arellano (2008) when $h(x_t) = 1$.

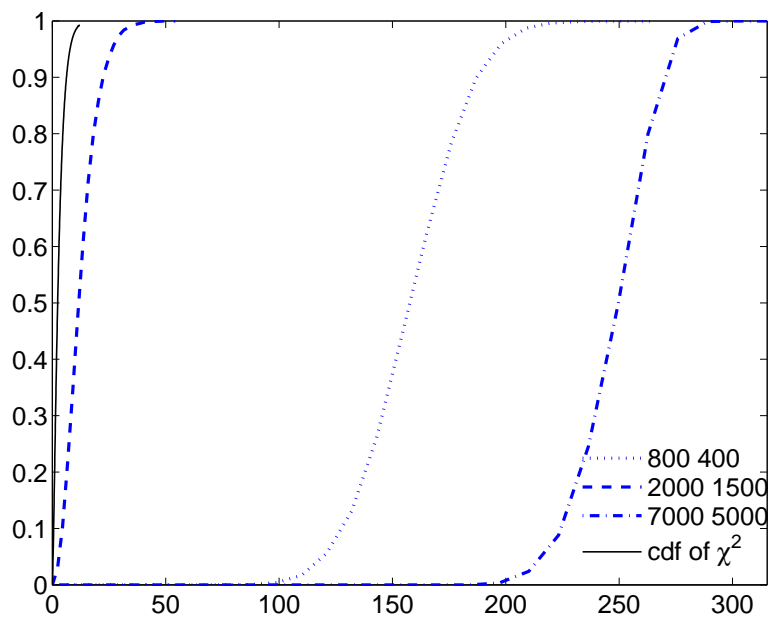


Figure 5: Cumulative distribution functions of the den Haan and Marcet's statistic in model I of Aguiar and Gopinath (2006) when the model is solved with discrete state space and for weights $h(x_t) = [1, y_t, b_t]$. The first (second) term in each label corresponds to the number of grid points for assets (endowment shocks).

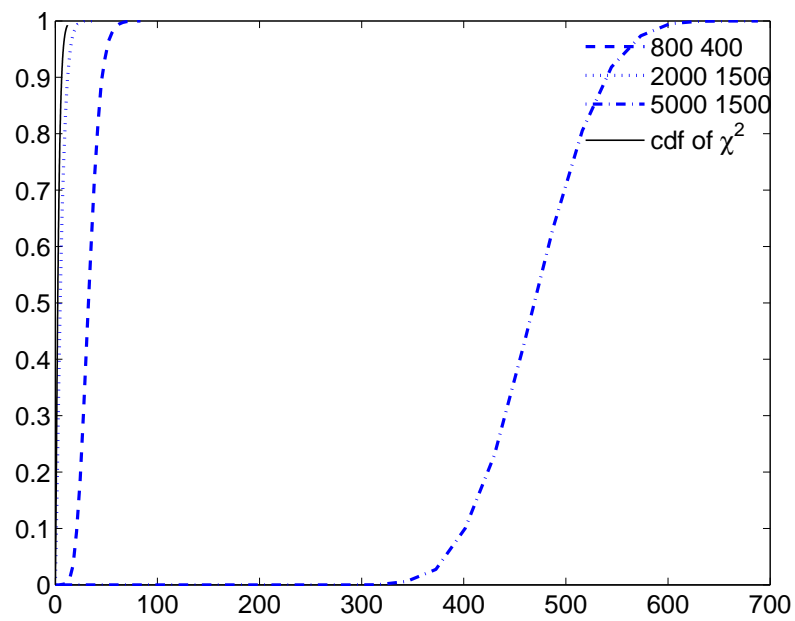


Figure 6: Cumulative distribution functions of the den Haan and Marcet's statistic in model II of Aguiar and Gopinath (2006) when the model is solved with discrete state space and for weights $h(x_t) = [1, y_t, b_t]$. The first (second) term in each label corresponds to the number of grid points for assets (trend growth shocks).

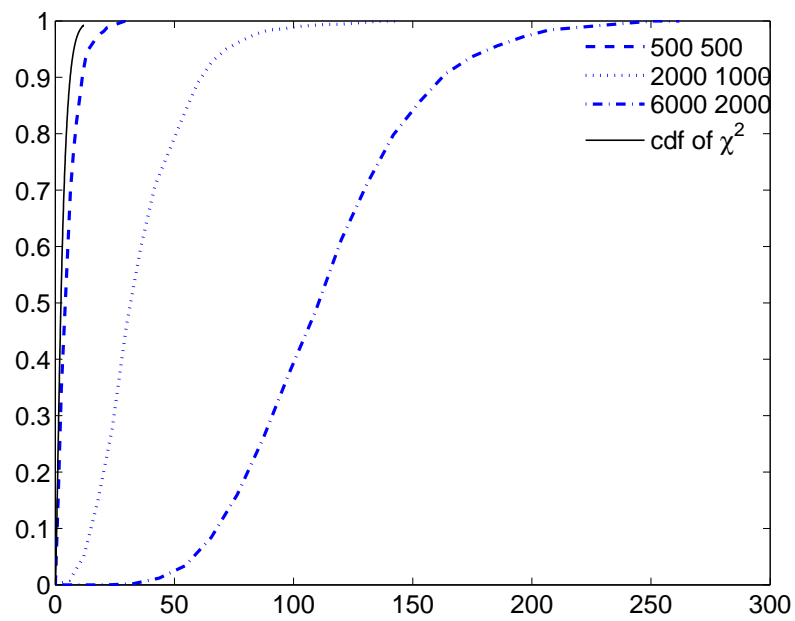


Figure 7: Cumulative distribution functions of the den Haan and Marcet's statistic in Arellano (2008) when the model is solved with discrete state space and for weights $h(x_t) = [1, y_t, b_t]$. The first (second) term in each label corresponds to the number of grid points for assets (endowment shocks).

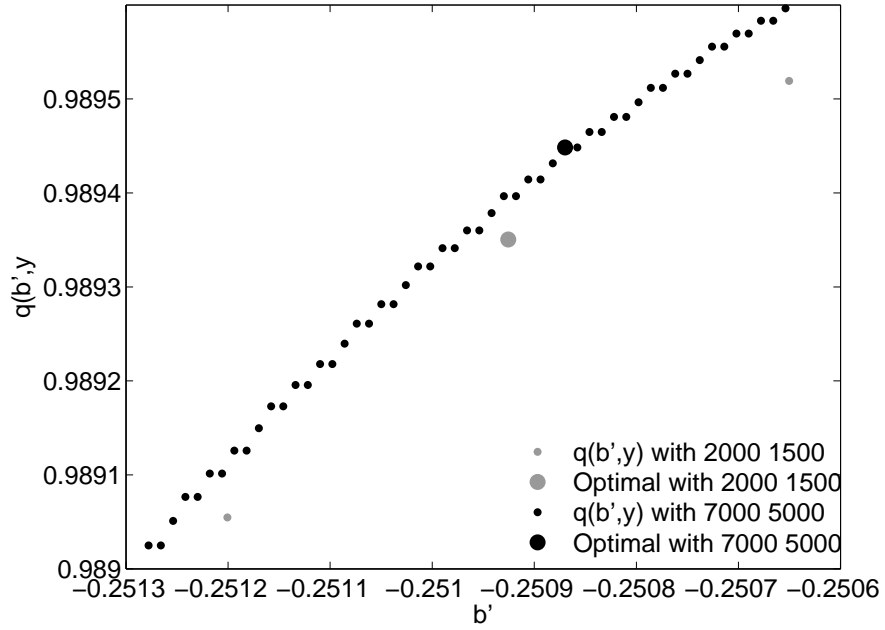


Figure 8: Bond price function faced by the agent and the optimal choice when Model I in Aguiar and Gopinath (2006) is solved using discrete state space but two different grid specifications. The graph was computed assuming that the initial endowment is equal to the unconditional mean and that the initial debt level equals the mean debt observed in the simulations. The evaluation of q_1 at the optimum when q_1 is approximated using $\Delta = 1$ produces a lower value for the finest grid specification.

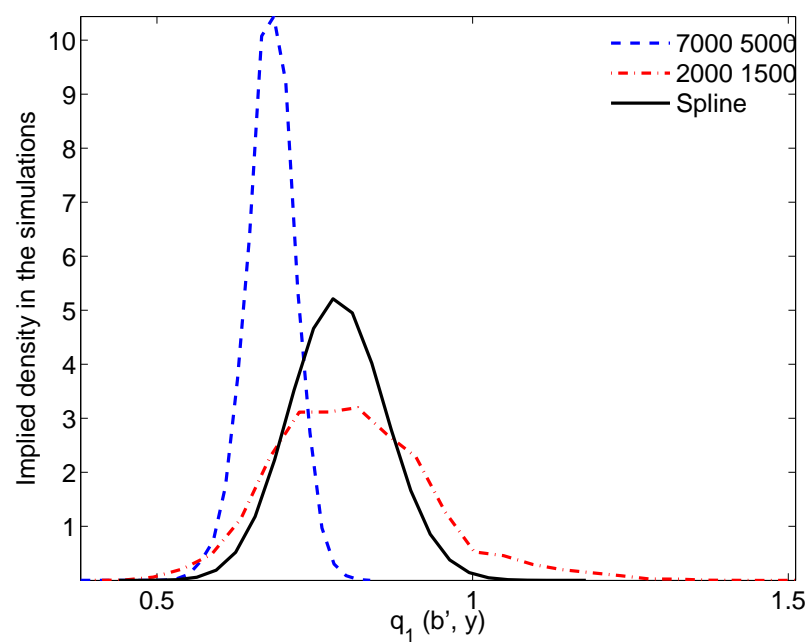


Figure 9: Density function of $q_1(b', y)$ in the simulations of Model I in Aguiar and Gopinath (2006). The density was computed using the same sample periods that were used to compute the den Haan and Marcet's statistic.

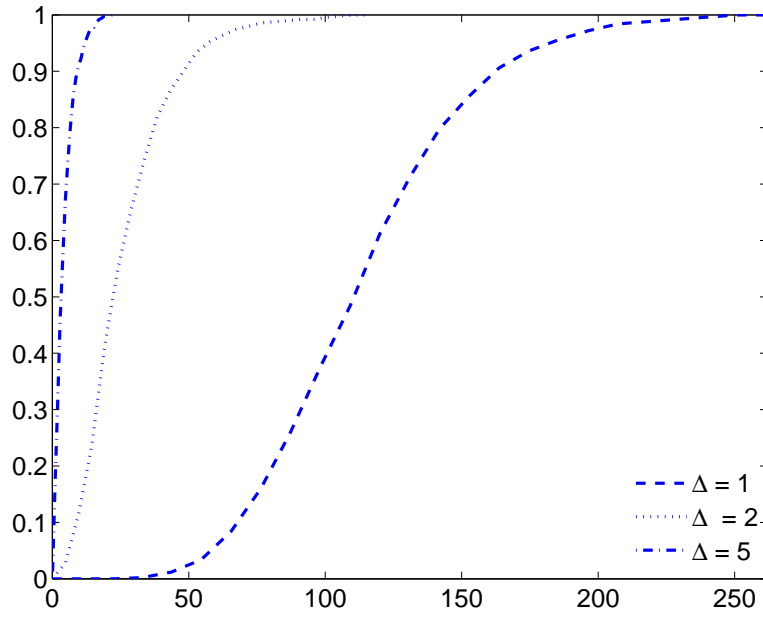


Figure 10: Cumulative distribution functions of the den Haan and Marcet's statistic in Arellano (2008) when the model is solved with discrete state space, our finest grid specification, and for weights $h(x_t) = [1, y_t, b_t]$. The graph shows that the distribution of the statistic varies substantially with the value of Δ .